Flexible Modes for Arithmetization-Oriented Compression Functions

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TU Wien Security & Privacy Group

Verifiable Computation, Blockchains, and ZK-SNARKs

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- Virtual Machines, Blockchains, Recursive SNARKs...

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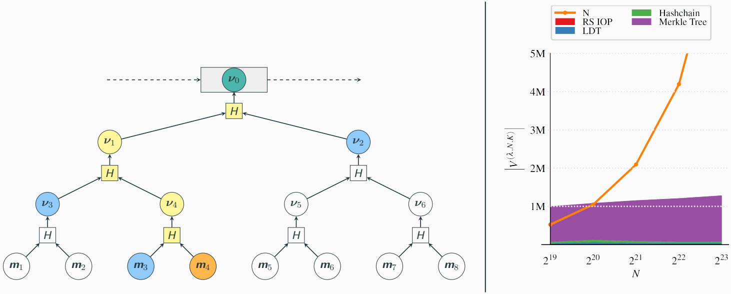

Figure 1: Left: binary Merkle Tree. Right: Fractal [12] verifier.

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- Native (SW/HW) performance is still important!

Flexible AO Compression Modes

Joint work E. Andreeva, R. Bhattacharyya, A. Roy

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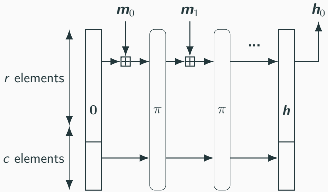

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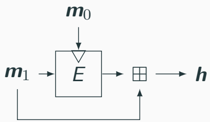

Davies-Meyer

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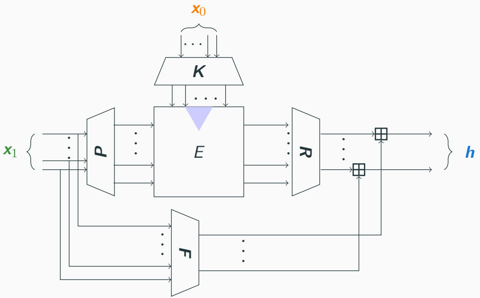

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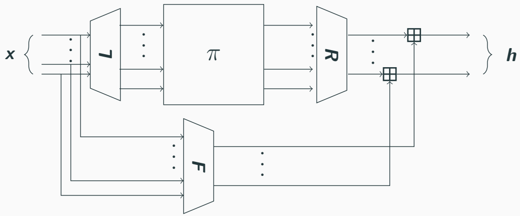

Security Results

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4. $\mathcal{A}$ can adaptively exploit partition imbalances.
5. Still, we obtain $\mathbf{Adv}_{\mathcal{C}}^{\mathrm{COL}}(q) \leq \frac{q^2+q}{p^{\ell}-q}$ ($\approx$ birthday attack).

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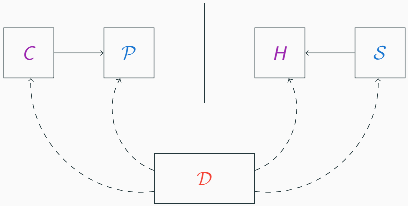

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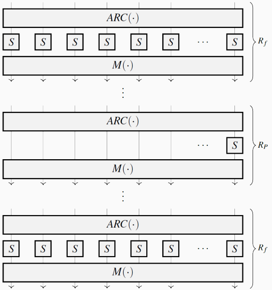

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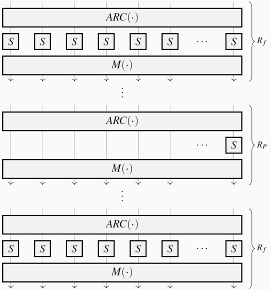

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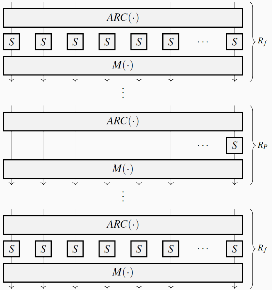

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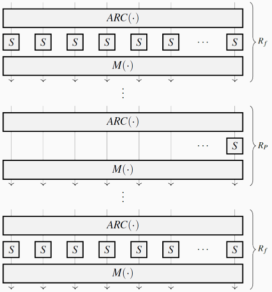

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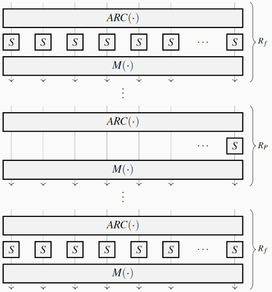

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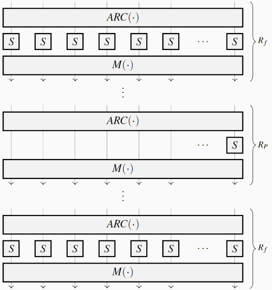

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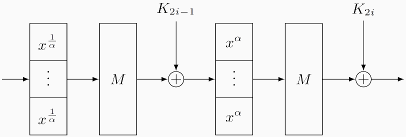

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- Security target: 128 bits of collision resistance.

Plain performance

Native execution performance:

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		$\log_2(p) \approx 256$			$\log_2(p) \approx 64$		
	Rate	LC-P	PGV	Sponge	LC-P	PGV	Sponge
HADES	2:1	7.52 µs	12.3 µs	13.2 µs	4.12 µs	2.57 µs	8.49 µs
	4:1	19.3 µs	12.1 µs	28.2 µs	14.8 µs	7.02 µs	35.0 µs
	8:1	69.7 µs	36.8 µs	84.4 µs	164 µs	27.5 µs	223.6 µs
Rescue	2:1	183 µs	385 µs	208 µs	22.1 µs	24.2 µs	33.3 µs
	4:1	217 µs	401 µs	220 µs	47.1 µs	43.9 µs	58.9 µs
	8:1	320 µs	458 µs	354 µs	136 µs	92.4 µs	143 µs

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Groth16 benchmarks

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		# R1CS constraints			Proof Generation time		
	Ratio	LC-P	PGV	Sponge	LC-P	PGV	Sponge
Hades	2:1	221	221	246	72.9 ms	73.0 ms	75.8 ms
	4:1	268	218	293	83.0 ms	73.4 ms	89.4 ms
	8:1	368	268	393	105 ms	83.9 ms	115 ms
Rescue	2:1	240	432	252	67.2 ms	107 ms	67.7 ms
	4:1	264	480	270	71.1 ms	116 ms	73.4 ms
	8:1	384	528	432	102 ms	126 ms	110 ms

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	Ratio	# gates LC-P	# gates PGV	# gates Sponge	Proof Generation time LC-P	Proof Generation time PGV	Proof Generation time Sponge
Hades	2:1	122	70	259	11.3 ms	11.1 ms	16.5 ms
	4:1	439	226	668	26.0 ms	16.2 ms	27.1 ms
	8:1	2065	847	2864	90.8 ms	47.5 ms	92.9 ms
Rescue	2:1	91	75	175	10.9 ms	8.58 ms	17.2 ms
	4:1	284	182	418	16.8 ms	11.5 ms	27.1 ms
	8:1	976	568	1213	47.9 ms	26.5 ms	49.0 ms

Merkle Tree arity benchmarks

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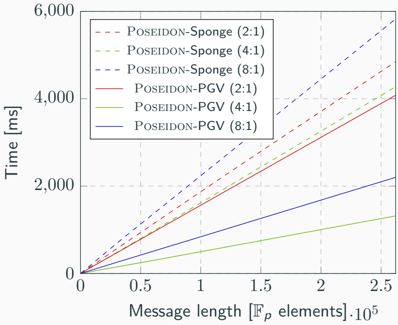

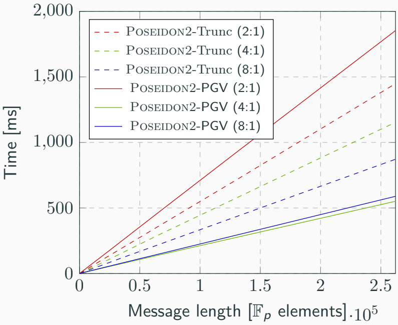

Roll credits

$\mathcal{T}he\ \ \mathcal{E}nd$

Thank you for your attention!

Any questions?

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