From Algebraic Geometry to Linear Cryptanalysis

Application to Anemoi

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(joint work with Tim Beyne)

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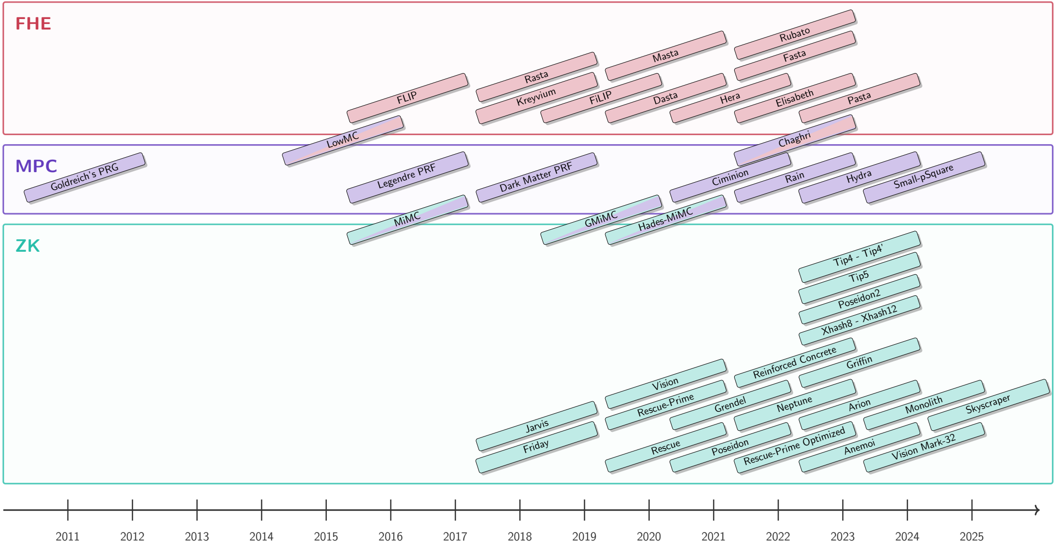
New symmetric primitives



A new context

Traditional case

Alphabet

Operations based on logical gates or CPU instructions.

$$\mathbb{F}_2^n, \text{ with } n \simeq 4, 8$$

Arithmetization-Oriented

Alphabet

Operations based on large finite-field arithmetic.

$$\mathbb{F}_q, \text{ with } q \in \{2^n, p\}, p \simeq 2^n, n \geq 32$$

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Cryptanalysis

Decades of cryptanalysis

- algebraic attacks ✓
- differential attacks ✓
- linear attacks ✓
- ...

Arithmetization-Oriented

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Operations based on large finite-field arithmetic.

$$\mathbb{F}_q, \text{ with } q \in \{2^n, p\}, p \simeq 2^n, n \geq 32$$

Cryptanalysis

≤ 8 years of cryptanalysis

- algebraic attacks ✓
- differential attacks ✗
- linear attacks ✗
- ...

Linearity

Definition

Let $\mathrm{F} : \mathbb{F}_q^n \to \mathbb{F}_q^m$ be a function and ω a primitive character. The **Walsh transform** for the character ω of the linear approximation (u, v) of F is given by

$$\mathcal{W}_{u,v}^{\mathrm{F}} = \sum_{x \in \mathbb{F}_q^n} \omega^{(\langle v, \mathrm{F}(x) \rangle - \langle u, x \rangle)} .$$

$$\boxed{\mathcal{W}_{u,v}^{\mathrm{F}} = q^n \cdot \mathcal{C}_{u,v}^{\mathrm{F}}}$$

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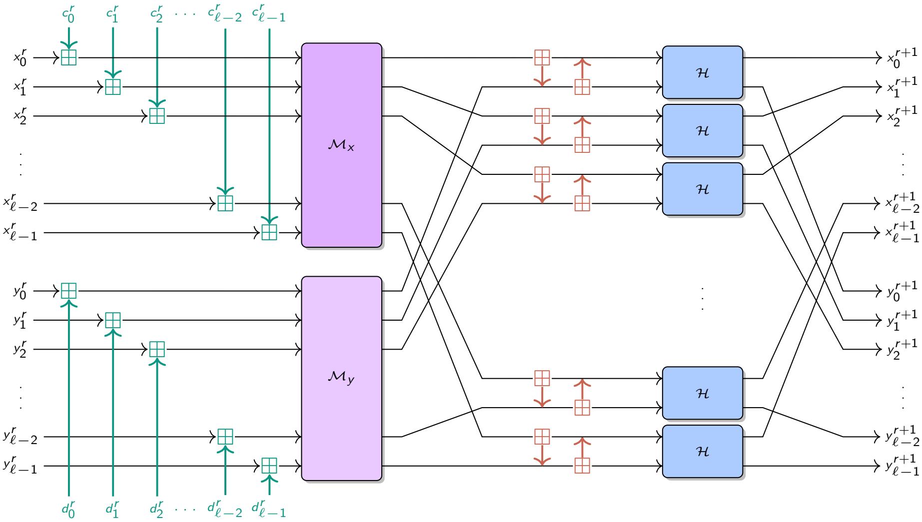
Definition

The **Linearity** $\mathcal{L}_F$ of $F : \mathbb{F}_q^n \to \mathbb{F}_q^m$ is the highest Walsh coefficient.

$$\mathcal{L}_F = \max_{u,v \neq 0} \left| \mathcal{W}_{u,v}^{F} \right| .$$

Anemoi round function

Introduced by [Bouvier, Briaud, Chaidos, Perrin, Salen, Velichkov and Willems, 2023].



Flystel - Definition

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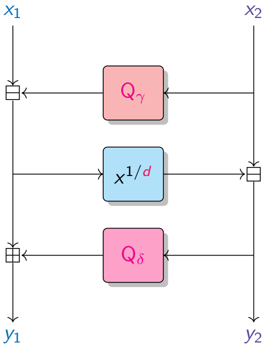
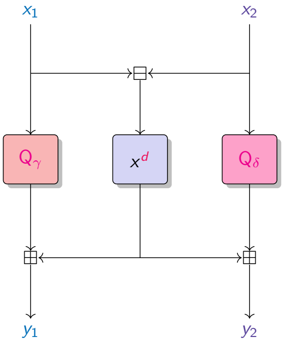


Open variant.



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$$\begin{cases} y_1 &= x_1 - Q_\gamma(x_2) + Q_\delta\left(x_2 - (x_1 - Q_\gamma(x_2))^{1/d}\right) \\ y_2 &= x_2 - (x_1 - Q_\gamma(x_2))^{1/d} . \end{cases}$$

$$\begin{cases} y_1 &= (x_1 - x_2)^d + Q_\gamma(x_1) \\ y_2 &= (x_1 - x_2)^d + Q_\delta(x_2) . \end{cases}$$

Closed Flystel in $\mathbb{F}_{2^n}$

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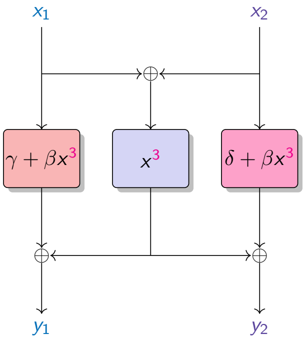

Closed Flystel.

$$\mathcal{L}_F = \max_{u,v \neq 0} \left| \sum_{x \in \mathbb{F}_{2^n}^2} (-1)^{(\langle v, F(x)\rangle - \langle u, x\rangle)} \right|$$

Bound

Linearity bound for the Flystel:

$$\mathcal{L}_F \leq 2^{n+1}$$

Closed Flystel in $\mathbb{F}_p$

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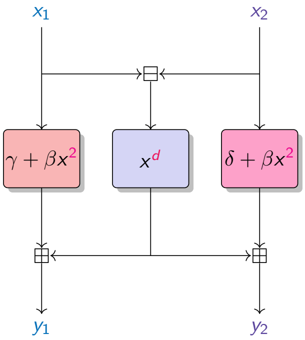

Closed Flystel.

d is a small integer s.t.
$x \mapsto x^d$ is a permutation of $\mathbb{F}_p$
(usually $d = 3, 5$).

$$\mathcal{L}_F = \max_{u,v \neq 0} \left| \sum_{x \in \mathbb{F}_p^2} e^{\left(\frac{2i\pi}{p}\right)(\langle v, F(x)\rangle - \langle u, x\rangle)} \right|$$

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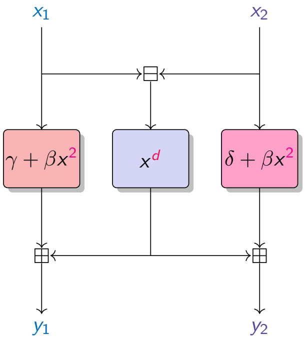

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How to determine an accurate bound for the linearity of the Closed Flystel in $\mathbb{F}_p$?

Weil bound

Proposition [Weil, 1948]

Let $f \in \mathbb{F}_p[x]$ be a univariate polynomial with $\deg(f) = d$. Then

$$\mathcal{L}_f \leq (d-1)\sqrt{p}$$

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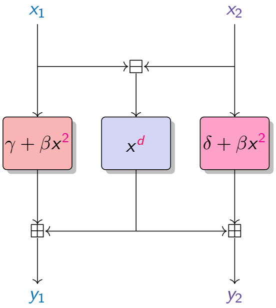


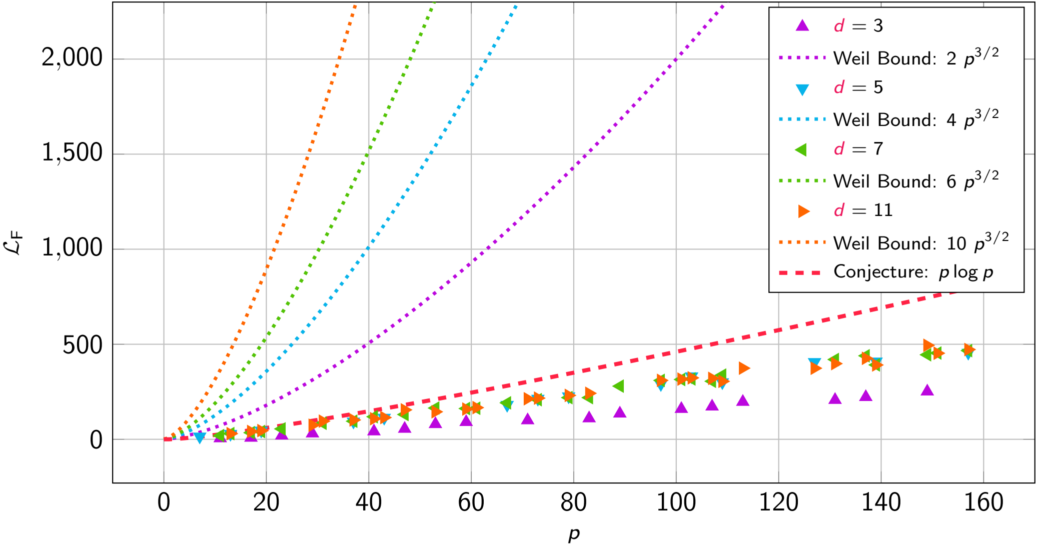
Closed Flystel.

$$\mathcal{L}_F \leq (d-1)p\sqrt{p}\ ? \qquad \begin{cases} \mathcal{L}_{\gamma+\beta x^2} & \leq \sqrt{p}\ , \\ \mathcal{L}_{x^d} & \leq (d-1)\sqrt{p}\ , \\ \mathcal{L}_{\delta+\beta x^2} & \leq \sqrt{p}\ . \end{cases}$$

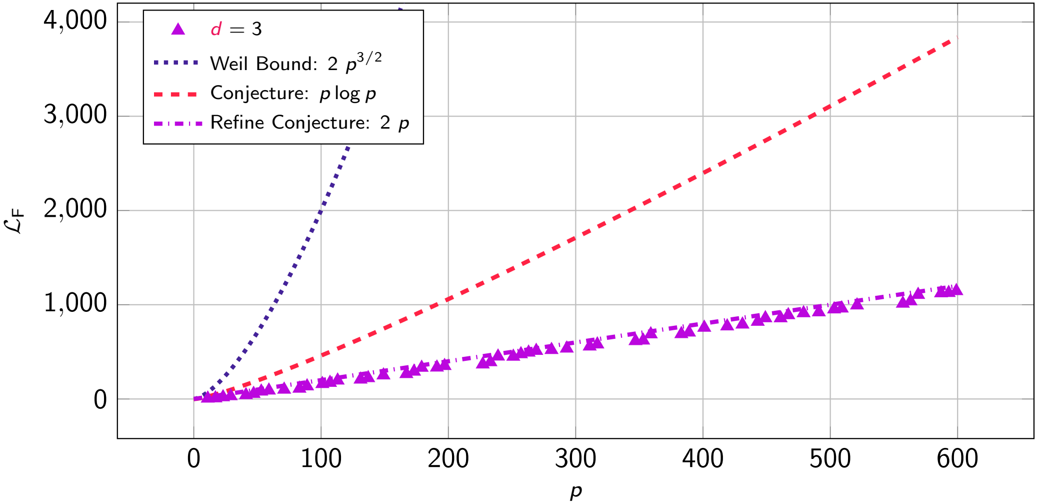
Conjecture

$$\mathcal{L}_F = \max_{u,v \neq 0} \left| \sum_{x \in \mathbb{F}_p^2} e^{\left(\frac{2i\pi}{p}\right)(\langle v, F(x)\rangle - \langle u, x\rangle)} \right| \leq p \log p$$

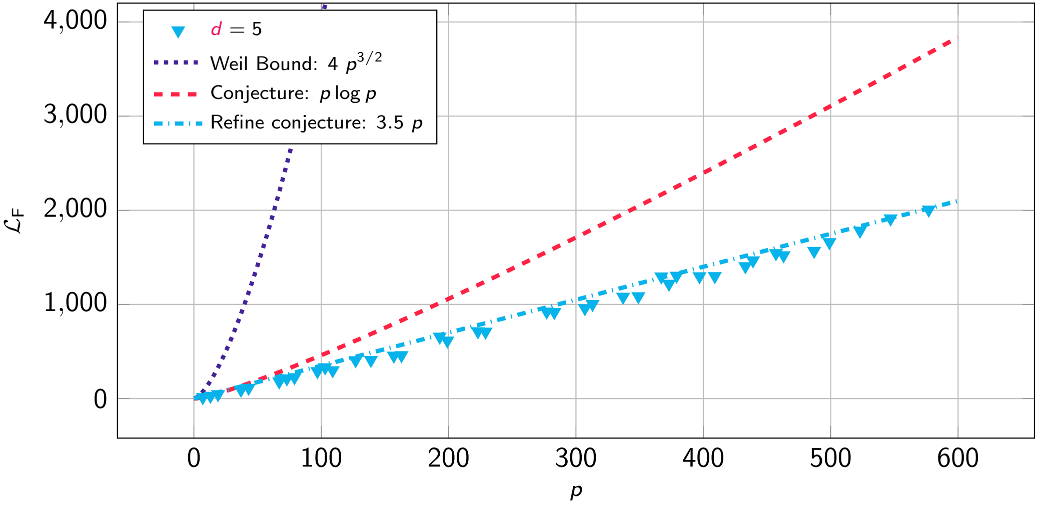
Experimental results



Experimental results ($d = 3$)



Experimental results ($d = 5$)



Take-away

AO primitives: new symmetric primitives defined over prime fields.

Need for new linear cryptanalysis tools

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$\boxed{\text{Need for new linear cryptanalysis tools}}$

This Talk:

- ⋆ Applications of results for exponential sums (generalization of Weil bound)

$$\mathcal{W}^{\mathrm{F}}_{u,v} = \sum_{x \in \mathbb{F}_q^n} \omega^{(\langle v, \mathrm{F}(x) \rangle - \langle u, x \rangle)} \quad \rightarrow \quad S(f) = \sum_{x \in \mathbb{F}_q^n} \omega^{f(x)} .$$

- ⋆ $\mathbb{F}_q$ is a finite field s.t. q is a power of a prime p.
- ⋆ Functions with 2 variables $\mathrm{F} \in \mathbb{F}_q[x_1, x_2]$.

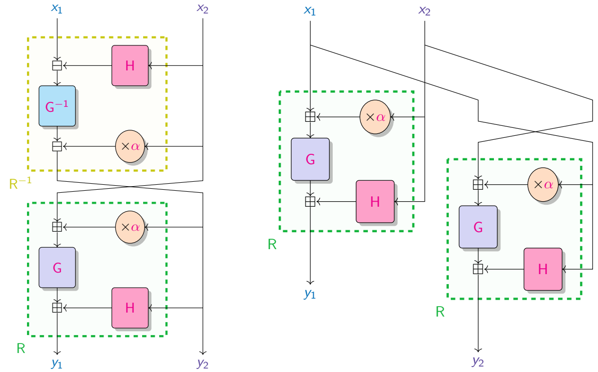
Generalizations of Weil bound

- ⋆ Deligne bound
  - ⋆ Application to the Generalized Butterfly construction
- ⋆ Denef and Loeser bound
  - ⋆ Application to 3-round Feistel construction
- ⋆ Rojas-León bound
  - ⋆ Application to the Generalized Flystel construction

Generalized Butterfly

Originally introduced by [Perrin, Udovenko and Biryukov, 2016] over binary fields, $\mathbb{F}_{2^n}^2$, n odd.

$\textsc{Butterfly}[G, H, \alpha]$, with $G : \mathbb{F}_q \to \mathbb{F}_q$ a permutation, $H : \mathbb{F}_q \to \mathbb{F}_q$ a function and $\alpha \in \mathbb{F}_q$.



Open variant. *Closed variant.*

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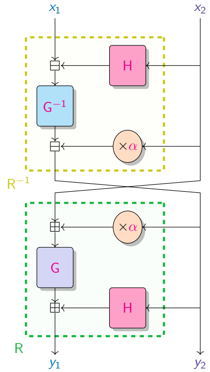
$$f(x_1, x_2) = \langle v, \mathsf{F}(x) \rangle - \langle u, x \rangle = v_1 \mathsf{G}(x_1 + \alpha x_2) + v_2 \mathsf{G}(x_2 + \alpha x_1) + v_1 \mathsf{H}(x_2) + v_2 \mathsf{H}(x_1) - u_1 x_1 - u_2 x_2 \ .$$



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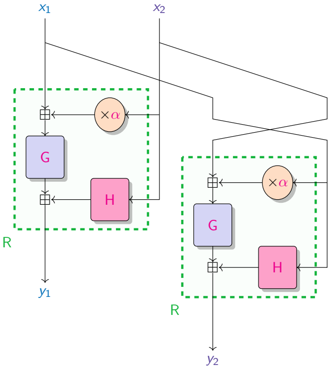


Closed variant.

Deligne Bound

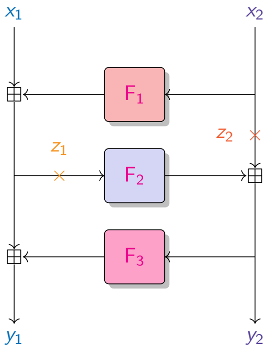
The hypersurface defined by $f_d = 0$ (the degree-d homogeneous component of f) is smooth so that

$$\boxed{\mathcal{L}_\mathsf{F} \leq (\max\{\deg \mathsf{G}, \deg \mathsf{H}\} - 1)^2 \cdot q}$$

3-round Feistel

Let $\textsc{Feistel}[F_1, F_2, F_3]$ be a 3-round Feistel network with F_2 a permutation and $d_1 \geq d_3$ where

$$d_1 = \deg(F_1), d_2 = \deg(F_2), \text{ and } d_3 = \deg(F_3) .$$



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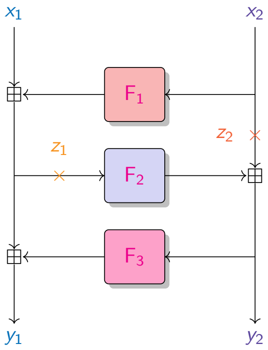
$$f(z_1, z_2) = \langle v, F(z) \rangle - \langle u, z \rangle = v_1 F_3(z_2 + F_2(z_1)) + v_2 F_2(z_1) + u_1 F_1(z_2) + (v_1 - u_1) z_1 + (v_2 - u_2) z_2 \ .$$

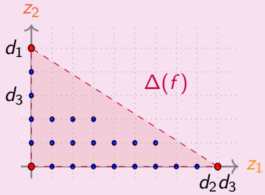


A 3-round Feistel.

Denef-Loeser Bound

f is commode and non-degenerate, with Newton number:

$$\nu(f) = (d_1 - 1)(d_2 d_3 - 1) \ .$$



$$\boxed{\mathcal{L}_F \leq (d_1 - 1)(d_2 d_3 - 1) \cdot q}$$

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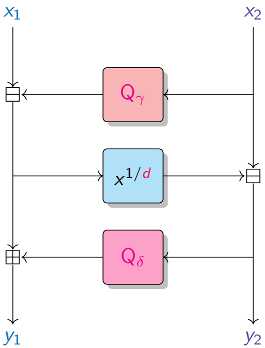

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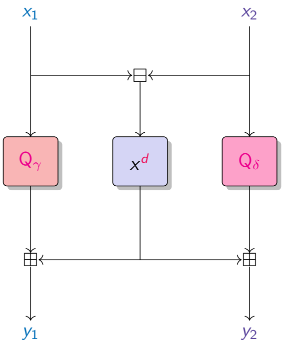

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$$\begin{cases} y_1 &= x_1 - Q_\gamma(x_2) + Q_\delta(x_2 - (x_1 - Q_\gamma(x_2))^{1/d}) \\ y_2 &= x_2 - (x_1 - Q_\gamma(x_2))^{1/d}\,. \end{cases}$$

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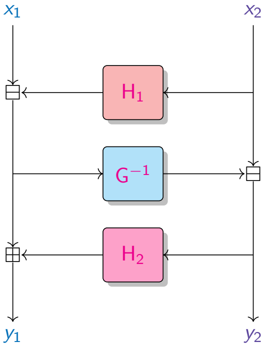
Generalized Flystel - Definition

$F = \textsc{Flystel}[H_1, G, H_2]$, with $G : \mathbb{F}_q \to \mathbb{F}_q$ a permutation, and $H_1, H_2 : \mathbb{F}_q \to \mathbb{F}_q$ functions.



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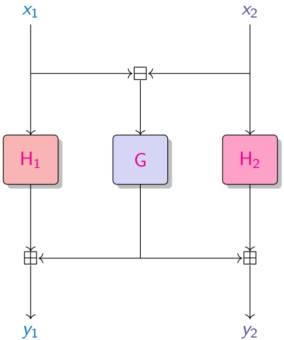

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$$\begin{cases} y_1 &= x_1 - H_1(x_2) + H_2(x_2 - G^{-1}(x_1 - H_1(x_2))) \\ y_2 &= x_2 - G^{-1}(x_1 - H_1(x_2))\,. \end{cases}$$

$$\begin{cases} y_1 &= G(x_1 - x_2) + H_1(x_1) \\ y_2 &= G(x_1 - x_2) + H_2(x_2)\,. \end{cases}$$

Isolated singularities

Definition

- ⋆ A singular point of a hypersurface is **isolated** if there exists a Zariski neighborhood of the point that contains no other singular points.
- ⋆ A polynomial g is **quasi-homogeneous** of degree δ is there exists $w_1, \ldots, w_n$ s.t.

$$g(\lambda^{w_1} x_1, \ldots, \lambda^{w_n} x_n) = \lambda^{\delta} g(x_1, \ldots, x_n) \ .$$

- ⋆ The **Milnor number** of the singularity is equal to $\prod_{i=1}^{n} (\delta / w_i - 1)$

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Example: Let $f(x) = (x-1)^d$.

- $x = 1$ is the only singular point of $f = 0$.
- Up to translation, we can consider the singularity in the origin: $g(x) = x^d$.

$$g(\lambda^w x) = (\lambda^w x)^d = \lambda^{w \cdot d} x^d = \lambda^{w \cdot d} g(x) \qquad \text{so that } \delta = w \cdot d$$

- Milnor number of the singularity: $\delta / w - 1 = d - 1$.

Rojas-León Theorem

Theorem [Rojas-León, 2006]

Let $f \in \mathbb{F}_q[x_1, \dots, x_n]$, s.t. $\deg(f) = d$.

Suppose that $f = f_d + f_{d'} + \cdots$, where f_d, $f_{d'}$, are resp. **the degree-d, degree-d', homogeneous component** of f, with $\gcd(d, p) = \gcd(d', p) = 1$ and $d'/d > p/(p + (p-1)^2)$.

If the following conditions are satisfied

- ⋆ the hypersurface defined by $f_d = 0$ has at worst **quasi-homogeneous isolated singularities** of degrees prime to p with **Milnor numbers** $\mu_1, \dots, \mu_s$,
- ⋆ the hypersurface defined by $f_{d'} = 0$ contains none of these singularities,

then we have

$$|S(f)| = \left| \sum_{x \in \mathbb{F}_q^n} \omega^{f(x)} \right| \leq \left((d-1)^n - (d-d') \sum_{i=1}^{s} \mu_i \right) \cdot q^{n/2} \ .$$

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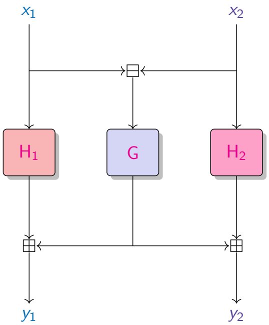
$\boxed{\text{Linearity bound for } n = 2\text{: } \mathcal{L}_F \leq ((d-1)^2 - (d-d')\sum_{i=1}^{s} \mu_i) \cdot q.}$

Generalized Flystel - Bound

Let $\mathsf{F} = \textsc{Flystel}[\mathsf{H}_1, \mathsf{G}, \mathsf{H}_2]$, with G a permutation, $\mathsf{H}_1, \mathsf{H}_2$ functions ($\deg \mathsf{G} > \deg \mathsf{H}_1, \deg \mathsf{H}_2$).

$$\begin{aligned} f(x_1, x_2) &= \langle (v_1, v_2), \mathsf{F}(x_1, x_2) \rangle - \langle (u_1, u_2), (x_1, x_2) \rangle \\ &= (v_1 + v_2)\, \mathsf{G}(x_1 - x_2) + v_1 \mathsf{H}_1(x_1) + v_2 \mathsf{H}_2(x_2) - u_1 x_1 - u_2 x_2 \ . \end{aligned}$$



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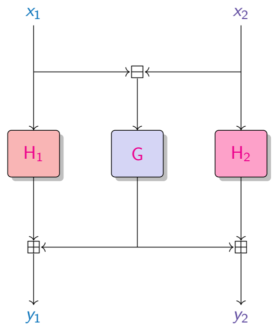

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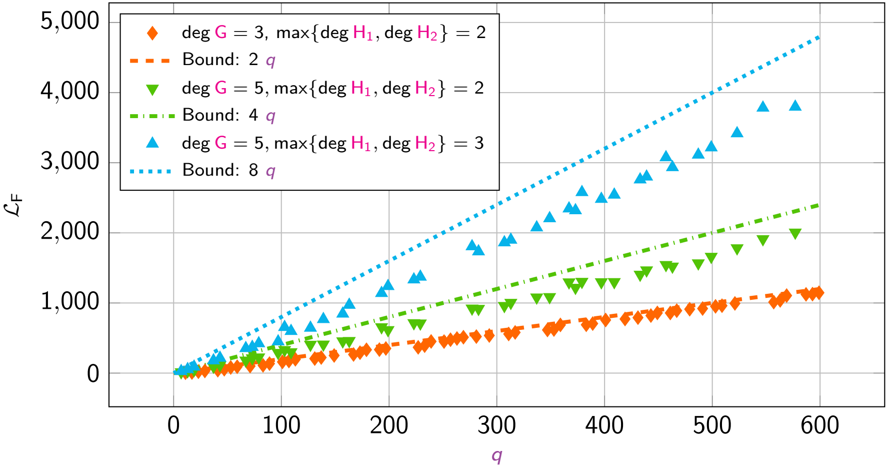
Linearity Bound

- ⋆ The hypersurface
$$f_d = (v_1 + v_2)(x_1 - x_2)^d = 0$$
contains one singular point $[1:1]$ of quasi-homogeneous type with Milnor number $d - 1$.
- ⋆ The hypersurface
$$f_{d'} = v_i x_i^{\deg \mathsf{H}_i} = 0$$
does not contain this point.

$$\boxed{\mathcal{L}_{\mathsf{F}} \leq (\deg \mathsf{G} - 1)(\max\{\deg \mathsf{H}_1, \deg \mathsf{H}_2\} - 1) \cdot q}$$

Generalized Flystel - Results

Let $F = \textsc{Flystel}[H_1, G, H_2]$ with H_1, G and H_2 monomials.



Low-degree permutations G, H_1 and H_2.

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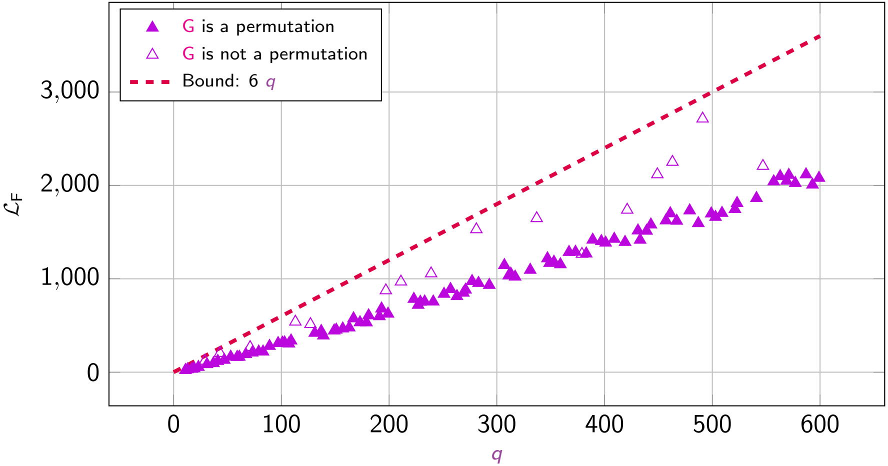

$\deg G = 7$ *and* $\deg H_1 = \deg H_2 = 2$.

Solving conjecture

Conjecture

Let $\mathsf{F} = \textsc{Flystel}[\mathsf{H}_1, \mathsf{G}, \mathsf{H}_2]$ be defined by $\mathsf{H}_1(x) = \gamma + \beta x^2$, $\mathsf{G}(x) = x^d$ and $\mathsf{H}_2 = \delta + \beta x^2$, with $\gamma, \delta \in \mathbb{F}_p$ and $\beta \in \mathbb{F}_p^\times$. Then

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Conjecture proved for $d \leq \log p$

Proposition

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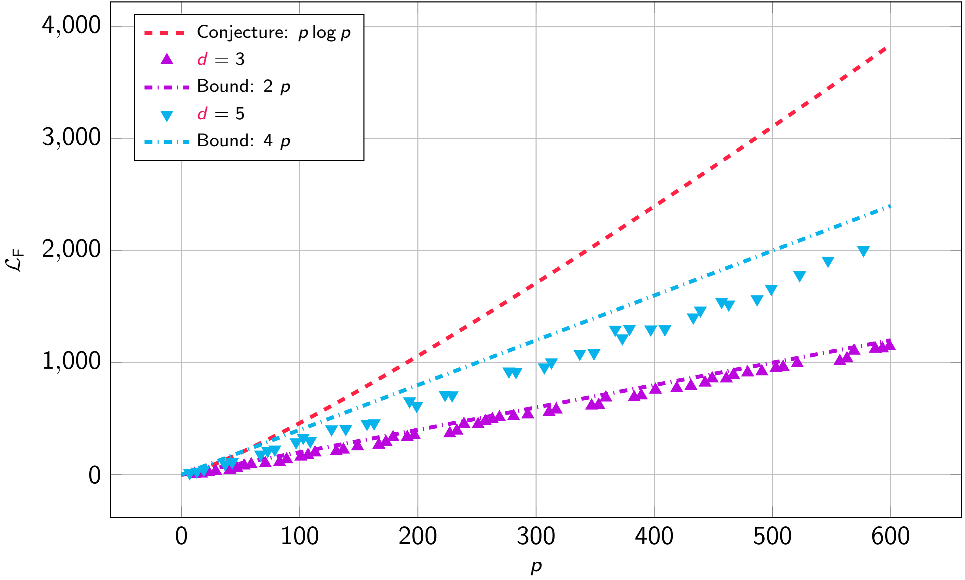
$$\mathcal{L}_\mathsf{F} \leq (d-1)p \ .$$

Solving conjecture



Cohomological framework

$$S(f) = \sum_{x \in \mathbb{F}_q^n} \omega^{f(x)} = \sum_{x \in \mathbb{F}_q^n} \omega^{(\langle v, \mathrm{F}(x) \rangle - \langle u, x \rangle)}$$

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$$|S(f)| = \left| \sum_{i=0}^{2n} (-1)^i \operatorname{Tr}\big(F \mid H_c^i(\mathbb{A}^n, \mathcal{L})\big) \right|$$

Sum of traces of the Frobenius automorphism on ℓ-adic cohomology groups.

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$$|S(f)| \leq \kappa \sum_{i=0}^{2n} \dim H_c^i(\mathbb{A}^n, \mathcal{L})$$

Conclusions

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- 3 different results... for 3 important constructions
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  - Rojas-León, 2006 — Generalization of the Flystel construction

$$\boxed{F \in \mathbb{F}_q[x_1, x_2], \ \exists C \in \mathbb{F}_q, \ \mathcal{L}_F \leq C \times q}$$

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Contribute to the cryptanalysis efforts for AOP.

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Thank you